Full-Wave Modeling of Coplanar Waveguide Discontinuities with Finite Conductor Thickness

Allen M. Tran, *Student Member, IEEE,* and Tatsuo Itoh, *Fellow, IEEE*

Abstract— **An extended version of the spectral domain approach (SDA) is developed to analyze discontinuities in open coplanar waveguide with finite metallization thickness. By making use of the exact Green's function in the spectral domain, the effects of surface wave and radiation phenomena are accurately accounted for. Both longitudinal and transverse components of the aperture electric fields are used in the analysis to allow modelling of structures with large transverse dimensions at high frequencies. The procedure also includes mode conversion near the discontinuities. As an illustration of the method, analytical steps and computed scattering parameters of the coplanar waveguide short-circuits and transitions will be provided and compared against measured data.**

I. Introduction

Recent emergence of the uniplanar configurations [1] and their constant increase in popularity at microwave and millimeter-wave frequencies in monolithic microwave integrated circuits (MMIC's) require accurate analysis of discontinuities appearing in the slot line and coplanar waveguide (CPW) family. In the majority of MMIC's developed so far, microstrip lines have been used as the main transmission line because their characteristics are well analyzed. On the other hand, CPW characteristics have not been investigated extensively and as a result no reliable design data exist. In the past several years, the operating frequencies of MMIC's have been increased steadily to reduce the size of the circuits. As these frequencies are increased, the characterizations of passive devices consisting of uniform transmission lines and discontinuity junctions are no longer sufficient by the so-called quasi-static analysis based on low frequency approximations; consequently, full wave analysis of solving Maxwell's equations is required. In addition, the conductor thickness effect in CPW and slot line, in general, is greater than that of microstrip lines because of the field configurations [2]; therefore, analytical procedures should take this effect into consideration.

In this paper, an extended version of the spectral domain approach [3] is utilized to characterize the uniform CPW with finite conductor thickness as shown in Fig. 1. The characteristics of the uniform CPW are then combined with the deterministic spectral domain approach [4] to analyze the discontinuities of the open CPW. The surface wave and radiation effects are automatically included in the formulation of the Green's function in the spectral domain. Theoretical results of different discontinuity configurations are compared against published and measured data to validate the accuracy of the proposed method.

Manuscript received September 20, 1992; revised February 1, 1993.
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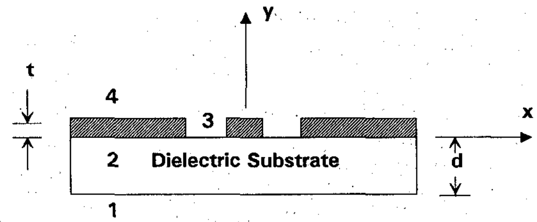


Fig. 1.

II. Analytical Formulation

The method consists of two main parts: eigenvalue formulation and discontinuity scattering-parameter calculation. The first part is based on the extended spectral domain approach presented by Kitazawa and Itoh in [5]; however, in our analysis, symmetrical CPW with isotropic and lossless substrate as shown in Fig. 1 will be assumed for simplicity. The electromagnetic fields in the open regions ($y < 0$ or $y > t$) are written in terms of Fourier integrals in the transverse x-direction as follows

$$E_{x,z}^{(m)}(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}_{x,z}^{(m)}(y)\cdot e^{-j\alpha x}\cdot d\alpha \cdot e^{-j\beta z}, \quad \text{where } m = 1,2,4 \tag{1}$$

while fields in the enclosed conductor region ($0 < y < t$) are expressed by the Fourier series representation with respect to the transverse x-direction in the following form

$$E_{x,z}^{(3)}(x,y,z) = \frac{1}{2W}\sum_{n=-\infty}^{\infty} \tilde{E}_{x,z}^{(3)}(y)\cdot e^{-j\alpha_n(x-x_0)}\cdot e^{-j\beta z}, \tag{2}$$

where β is the unknown propagation constant, x_0 represents the center of the aperture, and α_n is the Fourier variable determined such that the appropriate boundary conditions at the conductors are satisfied.

$$\alpha_n = \frac{2n\pi}{W}. \tag{3}$$

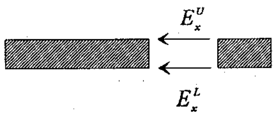


Fig. 2.

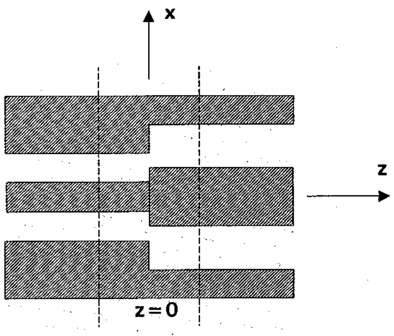


Fig. 3.

Introducing the unknown aperture fields $E^U(x,z)$ and $E^L(x,z)$ at $y=t$ and $y=0$, respectively, as shown in Fig. 2 and applying the continuity conditions of tangential fields at the interfaces $y=-d,0,t$, integral equations containing the unknown aperture fields $E^U(x,z)$, $E^L(x,z)$ and propagation β can be obtained:

$$E^{U,L}(x,z) = \left(E_x^{U,L}(x) + E_z^{U,L}(x)\right) \cdot e^{-j\beta z}. \tag{4}$$

To determine the unknown propagation constant and aperture fields, Galerkin's method [6] is applied in the spectral domain. Unknown components of the aperture fields are expressed in terms of known basis functions as follows

$$E_x^{U,L}(x') = \sum_{n=1}^{Nx} a_{xn}^{U,L} \cdot f_{xn}(x'), \tag{5}$$

$$E_z^{U,L}(x') = \sum_{n=1}^{Nz} a_{zn}^{U,L} \cdot f_{zn}(x'). \tag{6}$$

Substitute (5) and (6) into the integral equations, the determinantal equation for the propagation constant β is obtained. After β is determined, the aperture fields can in turn be computed. Therefore, the uniform CPW with finite conductor thickness has been characterized.

To compute the scattering parameters of the discontinuities, a modified version of the deterministic spectral domain method [4] is applied. The CPW structure with a discontinuity at $z=0$ is divided into three regions: the uniform region far away and to the left of the discontinuity, the discontinuity (or perturbed) region, and the uniform region far away and to the right of the discontinuity as illustrated in Fig. 3. Assume than an incident field originated from $z=-\infty$ propagates toward the discontinuity, then in the uniform region to the left of the discontinuity, the aperture fields can be expressed as the superposition of the incident field and a reflected field with an unknown complex reflection coefficient Γ

$$E_{x,z}^{U,L^{(\text{left})}}(x,z) = \sum_{n=1}^{Nx,Nz} a1_{x,z_n}^{U,L} \cdot f_{x,z_n}(x) \cdot \left(e^{-j\beta_1 z} \mp \Gamma \cdot e^{j\beta_1 z}\right), \tag{7}$$

while in the uniform region to the right of the discontinuity, the aperture fields only contain the transmitted field with an unknown complex transmission coefficient T

$$E_{x,z}^{U,L^{(\text{right})}}(x,z) = \sum_{n=1}^{Nx,Nz} a2_{x,z_n}^{U,L} \cdot f_{x,z_n}(x) \cdot T \cdot e^{-j\beta_2 z}. \tag{8}$$

In the perturbed region to the left of the discontinuity, the aperture fields are assumed to be the superposition of the field in the left uniform region and a perturbed field which contains

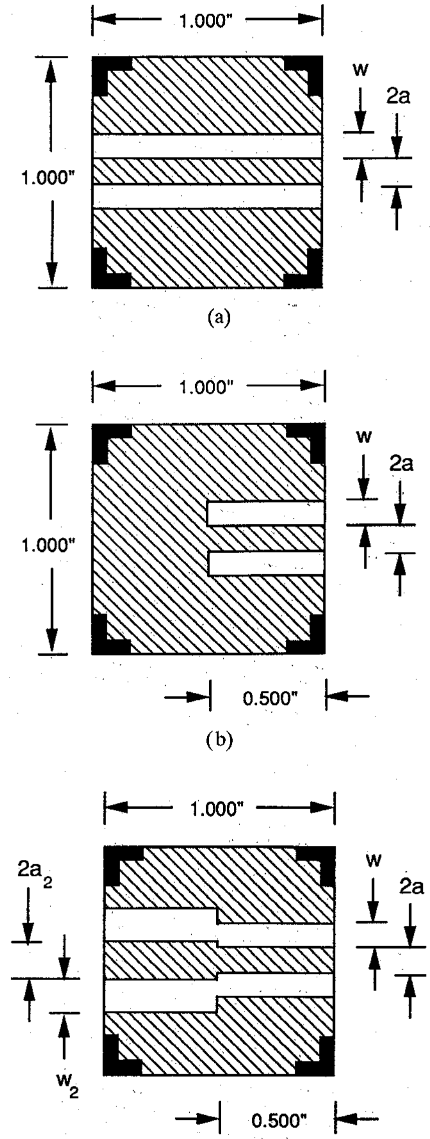
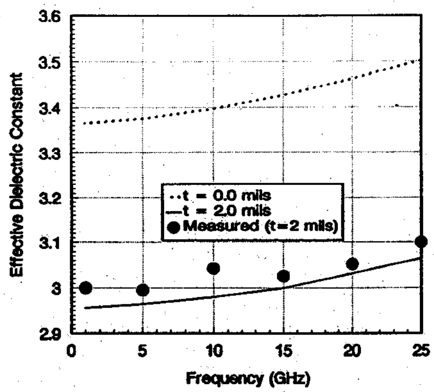


Fig. 4



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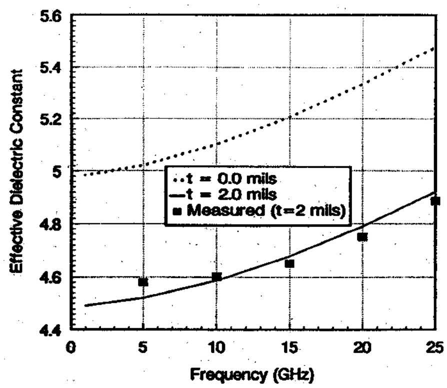
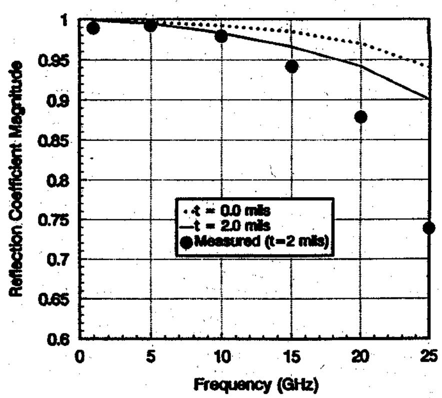


(b)

Fig. 5.



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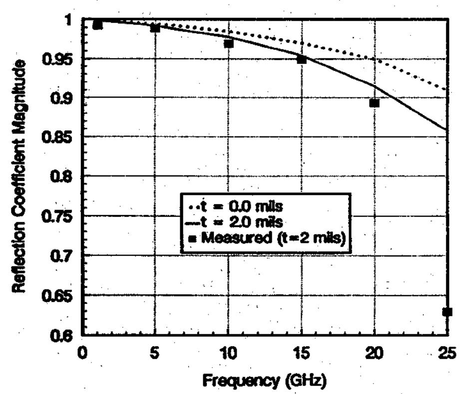


(b)

Fig. 6.

high order modes

$$E_{x,z}^{U,L^{(\text{disc,left})}}(x,z) = E_{x,z}^{U,L^{(\text{left})}} + \sum_{p=1}^{p1}\sum_{q=1}^{Q1} b1_{x,z_{pq}}^{U,L} \cdot g_{x,z_{pq}}(x) \cdot h_{x,z_{pq}}(z), \qquad (9)$$

where $g(x)$ and $h(z)$ are known basis functions.
Similarly, in the perturbed region to the right of the discontinuity, the aperture fields are assumed to be the superposition of the field in the right uniform region and a perturbed field which contains high order modes

$$E_{x,z}^{U,L^{(\text{disc,right})}}(x,z) = E_{x,z}^{U,L^{(\text{right})}} + \sum_{p=1}^{p2}\sum_{q=1}^{Q2} b2_{x,z_{pq}}^{U,L} \cdot g_{x,z_{pq}}(x) \cdot h_{x,z_{pq}}(z), \qquad (10)$$

The boundary conditions in the z direction are applied together with the method of moments in the spectral domain to generate a set of linear equations in which $\Gamma, \mathrm{T}, b1_{x,z_{pq}}$, and $b2_{x,z_{pq}}$ are unknowns. From the solutions of these unknowns, the aperture field distribution and scattering parameters can be determined.

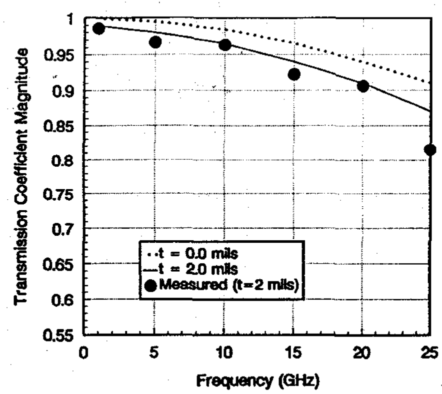
III. Basis Function

In the uniform CPW region, full domain basis functions $f_{x,z}(x)$ are used to satisfy edge conditions and also to provide fast convergence. These functions are modified Chebyshev polynomials of the first and second kinds defined in [3]. In the discontinuity region; however, subsectional basis functions $g_{x,z}(x)$ and $h_{x,z}(z)$ discussed in [6] are used to accommodate possible high variations in the field distribution and also to add flexibility to the structure configuration.

IV. Results

A FORTRAN program was written to compute the propagation constant β, the effective dielectric constant ε_{eff}, and the aperture field distributions of the uniform CPW. These information are then being used by another FORTRAN program to compute the scattering parameters and field distributions of the discontinuity structures such as short, open circuits, and transitions in CPW. Data presented later in this section were generated by these programs on a 386/PC computer. Typically, only two to four full-domain basis functions are required to characterize the uniform CPW and ten to twenty subsectional basis functions for accurate results in the discontinuity regions.

Since not much data have been published on discontinuities in open CPW, especially, with finite metallization thickness, six test circuits were fabricated to validate the theoretical results. These circuits included (2) uniform CPW lines, (2) CPW short-circuits, and (2) CPW transitions and were etched on 25-mil thick Duroid-6006 ($\varepsilon_r = 6.0$) and Duroid-6010



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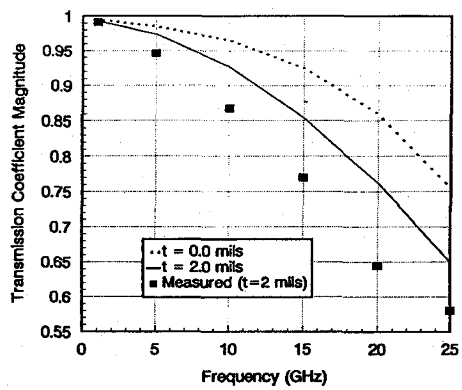
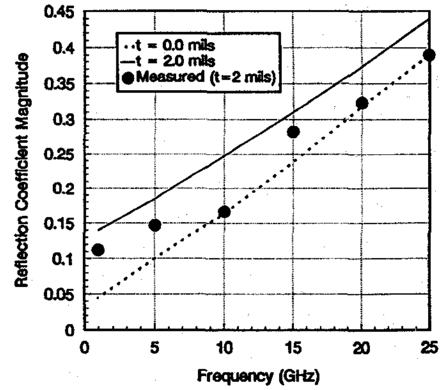


(b)

Fig. 7.



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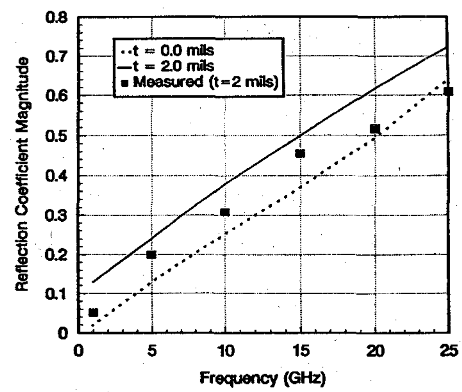

(b)

Fig. 8.

($\varepsilon_r = 10.2$) substrates with 2–mil thick copper. Each of the circuits was made on a 1-inch square substrate and tested using the Wiltron Universal Test Fixture connected to a Wiltron-360 Vector Network Analyzer. Since the Wiltron Universal Test Fixture with the K-connectors was advertised to work up to 26 GHz, only data from 1.0 GHz to 25.0 GHz will be presented here. Due to the lack of appropriate equipments and also dateline constraint, it was not possible to exclude the effects of the K-connectors and the K-to-CPW transitions from the measured data; therefore, the test results presented here include these effects. For that same reason, the measured phase data are meaningless and will not be compared. Dimensions of the test circuits are provided in Fig. 4 and Table I. Fig. 5 compares the predicted effective dielectric constants with measured effective constant for the circuits shown in Fig. 4(a). Data indicate good agreement between theoretical results and measurements for both $\varepsilon_r = 6.0$ and $\varepsilon_r = 10.2$ to within measurement errors. Fig. 5 also shows a large difference in effective dielectric constants of CPW structures with zero conductor thickness and with typical metallization thickness (2 mils); therefore, the effect of thick conductor cannot be ignored in coplanar waveguide. Computed and measured reflection coefficients of the test circuits in Fig. 4(b) are compared in Fig. 6. Data again show good agreement between measurements and predicted results, except at the higher frequencies where the K-connectors and their transitions to CPW degrade significantly. In addition, theoretical results do not include dielectric and conductor losses of the 1-in long CPW transmission line that occur in the measurements. These losses are usually large for Duroid materials at frequencies higher than 18 GHz; especially, for high dielectric constant materials. Radiation and surface wave losses are included in the computed data, but it is not possible to extract these losses from the measured data to compare with predictions. Predicted reflection coefficients for zero and finite conductor thicknesses are also compared in Fig. 6 to show the significant effect of conductor thickness. Finally, transmission and reflection coefficients of the CPW transitions described in Fig. 4(c) are presented in Figures 7 and 8. Computed results and measurements seem to agree well within experimental errors. Discrepancies are again due to the imperfections of the test fixture connectors and due to the losses in the CPW that are not accounted for in the theoretical calculations. Computed data for both thick and infinitesimally thin conductors are also shown in those same figures.

TABLE I
TEST CIRCUIT DIMENSIONS

ε_r	a	W	a_2	W_2
6.0	0.0125"	0.0091"	0.0175"	0.0105"
10.2	0.0125"	0.0143"	0.0225"	0.0189"

V. Conclusions

Open coplanar waveguide discontinuities with finite conductor thickness have been analyzed by using the extended spectral domain approach in conjunction with the deterministic spectral domain approach. Both components of the aperture electric fields are included in the formulation to allow accurate modelling of structures with large transverse dimensions. Surface wave and radiation phenomena are included in the Green's function formulation. Basis functions are chosen such that the edge conditions are properly satisfied together with a fast convergence rate. Numerical results include the effective dielectric constants of the uniform CPW and scattering parameters of the discontinuities. These results indicate good agreement between theory and measurements and also imply the effect of conductor thickness can not be ignored in the analysis of CPW.

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Allen M. Tran, photograph and biography not available at the time of publication.

Tatsuo Itoh (S'69–M'69–SM'74–F'92) for a photograph and biography, see this issue, p. 1482.